THE EFFECTS OF METACOGNITIVE TEACHING MODEL ON GRADE SIX STUDENTS' ATTITUDE TOWARDS MATHEMATICAL PROBLEM SOLVING

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Abstract

The main purpose of this study is to investigate the effectiveness of the metacognitive teaching model for teaching mathematics at the Middle School level in Myanmar. In this study, a mix-methods (QUAN $\rightarrow$ qual) design was adopted. The single-group pretest/posttest design, one of the designs of quasi-experimental research was adopted to collect the quantitative data and the case study design was applied to collect the qualitative data. It started in the first week of November 2021 and ended in the second week of January 2022. The study is geographically restricted to Yangon Region and a total of 129 Grade Six students participated. An attitude questionnaire and semi-structured interview questions were used as the research instruments. The data got from the attitude questionnaire were analyzed through descriptive statistics and Wilcoxon Signed Rank Test and the results showed that there were significant differences in the attitude of students towards problem solving before and after the study. According to the analysis of the responses of students and teachers to interview questions through the constant comparison method, it was found that the proposed model supported positive effects on problem solving behaviors of students, students' attitude towards problem solving and twenty-first century skills, and teachers' mathematics teaching. Additionally, qualitative findings supported the quantitative findings. Therefore, the research findings proved that the proposed metacognitive teaching model has a positive contribution to teaching problem solving at the Middle School level in Myanmar.

Keywords: Metacognition, Teaching Model, Mathematics, Problem Solving, Attitude towards Problem Solving

Introduction

Nowadays, exponential growth in science and technology has been demanding individuals with the skills necessary to find solutions to the problems in twenty-first century society. The Organization for Economic Co-operation and Development (OECD) (2014) stated that students who completed compulsory education should have sound mathematics skills and be able to apply these skills to solve the problems that they encounter in their lives.

In the school curriculum, mathematics is the best domain for problem solving and students need to become aware of their strengths, weaknesses, procedures, and strategies in problem solving (Jaye & Posamentier, 2006). Teaching problem solving through metacognition promotes self-directed learning and aims to develop individuals who are self-reliant and intrinsically motivated to seek the solution to any kind of problem (Zimmerman, 2000).

Research findings point out that students who have cognitive and metacognitive conflicts at the age of nine through eleven may develop negative emotions and poor motivation towards learning mathematics and which may interfere with their mathematical confidence and problem solving skills success for many years (Artino, 2009). Thus, teaching and learning mathematics should aim to develop students' ability and attitude in mathematical problem solving through metacognition.

Purpose of the Study

The main purpose of this study is to investigate the effectiveness of the metacognitive teaching model for teaching mathematics at the Middle School level in Myanmar.

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The specific objectives of the study are identified as follows:

1. To investigate the effects of the metacognitive teaching model on the attitude of the students towards mathematical problem solving
2. To examine the perception of students on solving mathematics problems through the metacognitive teaching model
3. To explore the opinions of teachers on teaching mathematics problems through the metacognitive teaching model

Research Questions

In this study, the three research questions are formulated as follows.

1. Is there a significant difference in the attitude of the students towards mathematical problem solving before and after the treatment?
2. What are the perceptions of the students on solving mathematics problems through the metacognitive teaching model?
3. What are the opinions of the teachers for using the metacognitive teaching model in teaching mathematics problems?

Scope of the Study

This study is geographically limited to Yangon Region. Participants in this study are Grade Six students and Mathematics teachers from the selected schools in (2021-2022) Academic Year.

Definitions of Key Terms

(i) Metacognition. Metacognition refers to thinking about thinking, and its main function is to plan, direct, control, examine, and evaluate all cognitive thinking processes; covering critical and creative thinking; to make appropriate decisions to solve a problem (Sang, 2003).

(ii) Teaching Model. A teaching model is an overall plan or pattern to learn specific kinds of knowledge, attitudes, and skills. It has a theoretical basis or philosophy behind it and encompasses specific teaching steps designed to accomplish desired educational outcomes (Joyce & Weil, 1972, as cited in Arends, 2007).

(iii) Mathematics. Mathematics is the investigating and use of patterns and connections in measures, space, and time (Ministry of Education [MoE], 2019).

(iv) Problem Solving. Problem solving is a multiple steps process where the problem solver must find relationships between the past experience (schema) and the problem at hand and then guide thinking directed towards the successful solution of a problem (Mayer, 1980).

(v) Attitude towards Problem Solving. Attitude towards problem solving refers to an individual emotion towards mathematical problem solving (either positive or negative), belief in mathematics, and also how that individual behaves towards problem solving (Hart, 1989, as cited in Ayob & Yasin, 2017).

Statement of the Problem

It can be easily seen that one of the problems encountered by Myanmar students while doing problem solving is that when they see a problem, they focus on getting the right answer and if the answer is right, they do not check, evaluate, and reflect on the whole process and they move to the next problems. However, if the answer is right by chance and the steps of the solution are wrong, this will lead to underachievement and lower motivation towards problem solving.

Another problem stated by Hardman, Stoff, Aung, and Elliott (2014) is that Myanmar teachers are using the transmission of knowledge and rote learning in teaching mathematics. Besides, creating opportunities for the cooperative and collaborative learning environments to promote critical thinking and a positive attitude towards problem solving is observed infrequently.

Significance of the Study

In Myanmar, mathematics is a compulsory subject in KG + 12 curriculum and the contents of mathematics are organized for students to understand new concepts and develop higher-order thinking skills and strongly focused on the development of students' thinking skills in problem solving. In mathematical problem solving, competent problem solvers are efficient at keeping track of what they know and how well or poorly their attempt to solve the problem (Jaye & Posamentier, 2006).

San Win (2010) also suggested that mathematics teachers should place an emphasis not on memorizing problem solving procedures but on developing students' thinking skills. Schraw (1998) and Applebaum (2015) also stated that when teachers themselves do not think critically or fail to share their thinking process through modeling and think-aloud in teaching mathematics problems, their students will be unlikely to develop mathematical thinking skills and positive attitudes towards problem solving.

Teaching that emphasizes mastering mathematics formulae seems questionable and wrong because it prevents students from understanding that mathematics can be meaningful. The way to solve the problem is that students need to be taught mathematical problem solving through metacognition. According to Kuiper (2002), metacognition supports reflective thinking, helps problem solving, gives responsibility, improves self-confidence for quicker decisions, and consequently develops positive attitudes towards problem solving.

In Mathematics classrooms, students often solve the problems superficially and produce the incorrect answers frequently (Nu Nu Nyunt & Aye Aye Myint, 2009). In order to improve achievement and positive attitude towards mathematics, students need to become aware of their strengths, weaknesses, typical behaviors, repertoire of procedures, and strategies in problem solving. Thus, mathematics teachers need to consider metacognition in the teaching-learning process not only for resulting the output but also for monitoring and controlling cognitive activities.

Review of Related Literature

Background Philosophical (Theoretical) and Psychological Considerations

All educational practices and research studies should be scientifically philosophic or philosophically scientific and the area of the investigation should be based on sound philosophical background (Khin Zaw, 2001). Thus, this study is mainly based on the following philosophical and psychological considerations.

In progressivism, learning is always an active process, the brain is not a passive receiver of knowledge but an active constructor of meaning through problem solving. According to this philosophy, mathematical problem solving involves reflection in action and reflection on action and these two concepts are closely related to metacognition (Schon, 1983).

From the cognitive perspective, learning is a change in a person's mental structure that provides the capacity to demonstrate different behaviors (Eggen & Kauchak, 1999). Cognitivists view that metacognition and problem solving are interrelated and these are the complex higher-order thinking skills in the human learning process. Metacognitive knowledge about problems and strategies and metacognitive skills of planning, monitoring, and evaluation are essential for successful problem solving (Gredler, 2001).

According to constructivism, to achieve the aim of effective learning, students need to plan their learning tasks, think of effective strategies to learn, as well as to evaluate, and make reflections on what they have learned. Constructivism also views learners as self-regulated and active participants in their learning (Churchill et al, 2013). Self-regulation also requires metacognitive mediators such as planning, monitoring, and evaluating (Schunk, 2012).

Gestalt theorists pointed out that the process of problem solving is based on the whole and the part relationship. They described that mathematics teachers should encourage both reproductive and productive thinking in the mathematics classroom with the caution of giving ready-made steps (Katona, 1940, as cited in Moslehpour, 1995).

Bruner (1964, as cited in Gredler, 2001) said that mathematical problem solving will be more effective and simpler by using symbols to represent abstract concepts and the think-aloud strategy can be used effectively in teaching students what and how to think about mathematical problem solving.

According to Piaget's developmental stages, Grade Six students fall into the formal operation stage and metacognition begins to develop during this stage and further in life (Flavell, 1977, as cited in Tarricone, 2011).

Besides, Vygotsky (1978) pointed out that metacognition develops from other regulations to self-regulation through modeling and scaffolding. Verbalization and internal verbalization of thought processes while doing problem solving support the development of metacognition. According to him, peer interaction, modeling, instructional scaffolding, verbalization of thought processes, and ZPD should be considered to develop metacognition in mathematical problem solving.

According to Bandura's social cognitive perspective on learning, metacognition is considered a sub-component of self-regulated learning in which, learning occurs through the interaction of personal, environmental, and behavioral factors (Bandura, 1986 as cited in Churchill et al., 2013). He also stated that cognitive modeling is one of the best strategies for demonstrating how to regulate cognition in mathematical problem solving and fosters the development of metacognition (Bandura, 1977, as cited in Gredler, 2001)

Theoretical Perspectives on Metacognition and Mathematical Problem Solving

Historically, the earliest philosopher who introduced the concept of metacognition in teaching is Buddha. According to scholars of Buddhism and Western psychologists, mindfulness (Samma-sati) is central to Buddhist teaching and has been closely associated with metacognition (Heys, Bang, Shea, Frith, & Fleming, 2020).

Mindfulness is a part of self-regulation and involves two related processes: monitoring and control (Schraw & Moshman, 1995). The metacognitive function of the right mindfulness includes not only observation and monitoring but also the skill of discrimination, refinement, and maintenance between thoughts, feelings, and behaviors (Ricketts, 2016).

In the realm of education and cognitive psychology, the first researcher who used the term metacognition was John Flavell (Zechmeister & Nyberg, 1982). Metacognition has been simply defined as thinking about thinking (Nazarieh, 2016). The main distinction between cognition and metacognition is that cognition is a constant flow of information (Langford, 1986) and metacognition is knowledge and awareness of processes and also the observance and management of such knowledge and processes (Flavell, 1979).

Looking back in the history of mathematics, since the 1980s, the studies conducted on mathematical problem solving have emphasized on if students are capable of monitoring their thinking process while solving problems and the term metacognition has been recognized as a key factor in problem solving (Ken, Clements, & Ellerton, 1996). In the context of mathematical problem solving, metacognition is the process used by students to analyze what they know about their learning, and to plan, monitor, evaluate, and modify the solution process (Bryce, Whitebread, & Szucs, 2014).

According to Flavell (1979), students can be trained to become aware of their cognitive processes through problem solving. Mayer (1998) also mentioned that successful problem solving depends on three components: (i) skills, (ii) meta-skills, and (iii) will. In these three components, metacognition in the form of meta-skill is central in problem solving because it monitors and controls the other components.

Goldberg and Bush (2003) also stated that teachers at all grade levels must learn how to develop and assess the metacognitive skills of their students to become proficient problem solvers in mathematics. Wilson and Conyers (2016) stated that the metacognitive approach to teaching directs students to focus their attention, monitoring their learning process and how well they are applying the selected strategies, and practicing the use of selective attention across contexts in the classroom and their personal lives.

To summarize, problem solving in mathematics is helpful in the proper development of one's mental power. No matter what types of problems are submitted, students who are competent problem solvers identify the problem, plan the strategy, ask themselves whether they are doing makes sense or not, adjust their problem solving strategies when necessary, and look back to reflect on the reasonableness of their solution and their approaches.

The Metacognitive Teaching Model for Problem Solving

This model is developed based on theoretical concepts of the information processing model, basic teaching model, psychological cybernetic model, algorithmic model, heuristic/plan generating model, multiplicity model, and multi-modal model. Additionally, the components of Brown's model, general problem solving model, Polya's problem solving model, and IDEAL problem solving model were taken into consideration.

The six components of metacognitive teaching model are as follows.

(i) Stimulating/Eliciting Domain-Specific Knowledge. Domain-specific knowledge is information that leads action to complete specific tasks. Thus, task-relevant prior knowledge to the student is elicited at the beginning of the lesson.

(ii) Informing Learning Outcomes. In the second stage, learning outcomes are informed to students to provide a set of shared expectations between the teacher and the students.

(iii) Presenting the Problem. In the third stage, a word problem from the prescribed textbook is presented to all students. This stage includes reading aloud, silent reading, and verbalization of the parts of the problem statement.

(iv) Solving the Problem through Explicit Modeling/Think-Aloud. This component is based on social cognitive theory and involves four stages: (i) identifying, (ii) planning, (iii) implementing, and (iv) evaluating. In each stage of the solution process, the teacher has to do explicit modeling through think-aloud. Five kinds of metacognitive questions: comprehension questions, connection questions, strategic questions, checking questions, and reflection questions are used to demonstrate what is going on in the teacher's head and how to monitor and control the thinking process while solving the problem.

(v) Consolidating in a Collaborative Setting. In this stage, students are formed into heterogeneous learning groups and they have to solve the problems by taking the role of thinker and listener. The thinkers have to explain their reasoning through verbalizing, while the listeners have to listen, record, ask questions, and make sure what the thinkers say. A set of metacognitive question cards are delivered to each group to help students to be aware of and monitor the problem solving phases.

(vi) Evaluating Performance and Transferring Learning. In this stage, students have to solve the problem independently. They have to do think-aloud about all the steps to monitor and control their thinking process. Next, they have to do reflective writing.

Method

The explanatory sequential (QUAN $\rightarrow$ qual), one of the basic mixed methods designs was adopted in this study**.**

Quantitative Research Method

Research Design. Jackson (2012) stated that the single-group pretest/posttest design is one of the variations of quasi-experimental research in which the two measures pretest (before) and posttest (after) can be compared and any differences in the measures are assumed to be the result of the treatment. Thus, the single-group pretest/posttest design was adopted to investigate the research question (1).

Subject. The sample size of the quantitative study is presented in Table 1.

Table 1 Population and Sample Size for Quantitative Study

No.	School	Population	Sample	Pretest	Treatment	Posttest
1	Practicing Middel School, Yankin	67	31	Attitude	MTM	Attitude
2	No. (5) BEHS, Mayangone	129	29	Attitude	MTM	Attitude
3	No. (2) EEHS, Thanlyin	79	36	Attitude	MTM	Attitude
4	No. (4) BEMS, Mingaladon	144	33	Attitude	MTM	Attitude
Total		419	129			

Note. BEHS = Basic Education High School; BEMS = Basic Education Middle School; MTM = Metacognitive Teaching Model

Instrument. The main instrument for quantitative study is an attitude questionnaire. The questionnaire is four points Likert scale and the items were categorized into three subscales: (i) emotion, (ii) belief, and (iii) behavior. There are eight items for each subscale, and a total of 24 items are included in the questionnaire. The internal consistency of the questionnaire was (Cronbach Alpha = .722).

Qualitative Research Method

Research Design. In qualitative research, case study design is a unique way of observing any natural phenomenon which exists in a set of data (Yin, 2003). Thus the case study design was adopted for research question (2) and (3).

Subject. In each school, students were assigned to groups A, B, and C. In each group, a student was elicited by using the random purposive sampling. Thus, a total of 12 students participated in the qualitative study. In addition, three Grade Six mathematics teachers also participated.

Instrument. The main instruments to collect data for the qualitative study are semi-structured interview questions for students and teachers.

The semi-structured interview questions for the students involve 17 questions and the items are divided into two categories: (i) problem solving behavior and (ii) perceptions on the metacognitive teaching model. The semi-structured interview questions for the teachers include

18 questions and the items are divided into two categories: (i) opinions on problem solving behavior of students and (ii) opinions on the metacognitive teaching model.

Study Procedure

This study started in the first week of November 2021 and ended in the second week of January 2022. The duration of the study was taken about eleven weeks. The pilot study was conducted with 25 Grade Six students from No. 2 Basic Education Middle School, Yankin.

Before the treatment period, students from each selected school were administered the attitude questionnaires. Then, they were taught problem solving through the stages of the metacognitive teaching model.

At the beginning of the lesson, students' domain-specific knowledge needed to solve the particular problems was elicited through different interaction patterns. Then, they were informed of the learning outcomes, and a word problem was presented. They were instructed to read the problem aloud and then silently, underline the keywords, and verbalize the parts of the problem statements to get a clear understanding of the problem context.

Then, the teacher's explicit modeling through thinking aloud was continued by four stages: (i) identifying, (ii) planning, (iii) implementing, and (iv) evaluating. In each stage of the solution process, five kinds of metacognitive questions: (i) comprehension question, (ii) connection question, (iii) strategic question, (iv) checking question, and (v) reflection question were used through thinking aloud.

Next, students solved the subsequent problems in a collaborative setting by taking the role of thinker and listener alternately. Throughout the solution process, all learning groups received metacognitive question cards.

After solving the problem in the collaborative setting, all students were instructed to solve the subsequent problems individually by verbalizing the solution process. Then, they were asked to do reflective writing based on what went well, what did not go well in the solution process, and what they would do next. After the treatment period, students were administered the attitude questionnaire.

Before the end of the study period, semi-structured interviews were conducted to explore the perspectives of students on solving problems through the metacognitive teaching model and to elicit teachers' opinions on teaching problem solving through the metacognitive teaching model.

Data Analysis

According to Pallant (2013), Wilcoxon Signed Rank Test can be used to measure the participants under two different conditions and the pretest results can be compared with the posttest results. Thus, the quantitative data were analyzed through descriptive statistics and Wilcoxon Signed Rank Test already installed in the Statistical Package for the Social Science (SPSS) version (24). In the qualitative data analysis, the constant comparison method is a very common type (Gay & Airasian, 2003). Thus, the qualitative data were analyzed through the constant comparison method.

Findings

Quantitative Research Findings

1. Findings of Students' Attitude towards Mathematical Problem Solving

Table 2 describes the comparison of the descriptive statistic results for students' responses to each item in the attitude questionnaire before and after the treatment.

Table 2 Descriptive Statistics Results for Students' Responses to Attitude Questionnaire

Emotion		**Before**		**After**	
No.	**Statement**	***M***	***SD***	***M***	***SD***
1	Have good feelings about learning mathematics	2.96	0.66	3.42	0.55
2	Solving mathematics problems is fun	2.99	0.74	3.34	0.53
3	Do not like sharing ideas with classmates	2.81	0.94	3.42	0.58
4	Think mathematics is a boring subject	3.13	0.92	3.53	0.58
5	Enjoy thinking aloud while solving problems	2.67	0.84	3.16	0.63
6	Enjoy to listen the thoughts and ideas of classmates	2.78	0.87	3.19	0.62
7	Enjoy sharing thoughts and ideas with classmates	3.02	0.79	3.29	0.61
8	Feel nervous to solve problems	2.86	0.89	3.40	0.58
Belief		**Before**		**After**	
No.	**Statement**	***M***	***SD***	***M***	***SD***
9	Mathematics is useful in real life situation	3.46	0.65	3.61	0.52
10	Feel confident in solving any kinds of problems	2.79	0.75	3.23	0.49
11	Mathematics is easy subject	2.45	0.80	3.01	0.53
12	Most problems are too hard to solve	2.42	0.84	3.10	0.59
13	Mathematics is useful only for exam	2.75	1.06	3.44	0.62
14	Good at giving reasons in solving problems	2.87	0.67	3.29	0.53
15	Can use mathematics to solve real life problems	2.82	0.80	3.16	0.57
16	Have confident only numerical calculation	2.19	0.82	3.09	0.63
	Behavior	**Before**		**After**	
No.	**Statement**	***M***	***SD***	***M***	***SD***
17	Underline the key points while reading	2.79	0.73	3.47	0.54
18	Divide the problem statements into different forms	2.95	0.77	3.40	0.53
19	Do not check on answer	2.99	0.85	3.66	0.55
20	Write down any answer	2.96	0.93	3.68	0.54
21	Give up if do not get the answer	3.07	0.89	3.56	0.55
22	Go through the solution and check if any mistakes	3.16	0.78	3.55	0.51
23	Try to find out the causes of mistakes	3.00	0.76	3.42	0.55
24	Find out different ways	2.84	0.71	3.29	0.49

Note. Number of Students = 129; Cronbach Alpha = .722.

According to the results of the descriptive statistics described in Table 2, it was found that the mean scores of each item before the treatment were increased after the treatment.

2. Findings of Wilcoxon Signed Rank Test Results on Attitude Questionnaire

Table 3 shows the Wilcoxon Signed Rank Test results for students' responses to the attitude questionnaire before and after the treatment.

Table 3 Wilcoxon Signed Rank Test Results for Students' Responses to Attitude Questionnaire

No.	**School**	**Pair**	***N***	***M***	***SD***	***Md***	***Wilcoxon (z) test***	***p***	***r***
1	Practicing Middel School, Yankin	Before	31	66.67	6.95	67	-4.63	.000***	.83
		After	31	77.64	5.10	77			

No.	School	Pair	*N*	*M*	*SD*	*Md*	*Wilcoxon (z) test*	*p*	*r*
2	No. (5) BEHS, Mayangone	Before	29	69.24	11.58	70	-4.67	.000***	.87
		After	29	80.17	7.00	79			
3	No. (2) EEHS, Thanlyin	Before	36	70.50	7.50	71	-5.16	.000***	.86
		After	36	81.36	6.34	81			
4	No. (4) BEMS, Mingaladon	Before	33	68.36	6.55	71	-5.02	.000***	.87
		After	33	83.51	4.58	84			

Note. BEHS = Basic Education High School; BEMS = Basic Education Middle School.
***$p < .001$.

The results in Table 3 point out that there were significant differences in the experimental group students' responses to the attitude questionnaire before and after the treatment.

Qualitative Research Findings

1. Findings on Perceptions of the Students on Solving Mathematics Problems through The Metacognitive Teaching Model

The perceptions of students on solving mathematics problems through the metacognitive teaching model were found follows.

(i) Problem Solving Behaviors

- They usually read the problem aloud and then read silently.
- They did underlining the keywords in the problem and paraphrased.
- They wrote givens and what was asked for.
- They performed drawing to visualize the context of the problem.
- They often explored the solution by comparing the similarities and differences between the problems previously solved and the problem at hand.
- They arranged the computation steps and did draft calculations.
- They usually took monitoring on computation steps.
- They usually evaluated the answers, units, numbers copied from the problem, computation steps, and whether the solution was relevant to the problem context.
- They took reflective writing after solving the problems.

(ii) Perceptions on Metacognitive Teaching Model

- Solving mathematics problems through the proposed model could change their feelings to become positive towards problem solving.
- They became confident in solving word problems.
- They had got the habit of controlling, monitoring, and taking reflection.
- They had developed twenty-first century skills.
- Metacognitive question cards gave clear instructions and helped them instead of the teacher.
- They wanted to learn problem solving through the teaching-learning procedure of the proposed model in further grades.
- Sharing reflective writing with the whole class and finding solutions through heuristic strategies could be enjoyable in doing problem solving.

2. Findings on Opinions of the Teachers for Using the Metacognitive Teaching Model in Teaching Mathematics Problems

The opinions of teachers on teaching problem solving through the metacognitive teaching model were found as follows.

(i) Problem Solving Behavior of Students

- All students read the problem aloud. Then, they continued silent reading and underlined the keywords given by the problems.
- They did paraphrasing to get a clear understanding of a problem.
- They looked back at the previously solved problems and analyzed the similarities and differences between problems and the given problem.
- They found the solution through drawing, arranged the computation steps, and did the draft calculations.
- They performed monitoring on writing givens, numbers, and calculation steps through thinking aloud.
- They checked the answers, units, numbers copied from the given problem, and computation steps.
- They wrote reflective writing based on what went well, what did not go well, and what they would do next.

(ii) Opinions on the Metacognitive Teaching Model

- Teaching problem solving through the metacognitive teaching model could solve the underachievement problem in mathematics due to careless mistakes.
- Solving problems within groups through metacognitive question cards could change students' attitudes to become positive towards problem solving.
- Students got the habit of controlling, monitoring, and self-reflection.
- Twenty-first century skills could be trained through the proposed model.
- The proposed model was relevant to all grades at the Middle School level.
- They had learned metacognition is an important concept in teaching problem solving.
- The proposed model had positive effects on their mathematics teaching.

Discussion

Research question (1) addressed the students' attitude towards problem solving before and after the treatment period and the results showed that there were significant differences in the attitude of the students before and after the treatment in all selected schools.

This finding is consistent with Cardelle-Elawar (1995) who examined Middle School students' performance in problem solving and found that students who have got metacognitive training are significantly improved in their attitude towards problem solving compare to students who got traditional teaching in the control condition. Correspondingly, the result supports to Kendir and Sahin (2013) who examined the effect of using metacognitive strategies for problem solving in geometry on Grade five students' achievement, metacognitive skills, and attitude and they discovered that students in the experimental group have developed better attitudes towards solving geometry problems.

Research question (2) and (3) explored the perceptions of the students and the opinions of teachers on the metacognitive teaching model. According to the data analysis results on students' responses to interview questions, the results were found that the metacognitive teaching model had effects on their problem solving behaviors, they had developed positive attitude towards problem solving, and they had developed twenty-first century skills. The findings on teachers' responses to interview questions showed that they advocated that the proposed metacognitive teaching model could change the problem solving behavior of students, could solve the underachievement problem of students in mathematics due to careless mistakes, and could support the students' attitudes to become positive towards problem solving and twenty-first century skills.

According to these findings, it was found that students' responses to interview questions were consistent with teachers' responses. These findings support to Schoenfeld (1987) who said that students who are trained to stop periodically during problem solving and ask themselves questions are able to control and reflect on the problem solving process and consequently improve their performance and attitude. In addition, these findings also advocate to Bandura's social cognitive learning theory in which a student could imitate certain skills from the teacher's skillful demonstration as his model and can master the skills if he acquires satisfaction and appropriate reinforcement (Gredler, 2001). These findings also acknowledge the constructivism in which learning is enhanced when individuals are socially participated in the learning process and engaged in new learning with prior knowledge and engaged in authentic learning tasks (Eggen & Kauchak, 1999).

Suggestions

In mathematics, problem solving is the core of the subject and it helps individuals to tackle the problems in their lives with confidence. Practically, to encourage a positive attitude towards problem solving, through metacognition, mathematics teachers should be familiar with teaching with and for metacognition. Teaching with metacognition is similar to reflection in action and teaching for metacognition looks like reflection on action.

The metacognitive approach to teaching emphasizes more on heuristics and strategies, not on algorithms and tactics (Biggs & Telfer, 1987). In the KG + 12 curriculum, starting from Grade two, some of the heuristic strategies such as drawing and finding patterns have been introduced to explore the solutions to particular problems. Therefore, mathematics teachers at the Middle School level should be familiar with heuristic strategies and metacognitive strategies such as explicit modeling, think-aloud, self-questioning, think-pair-share, and know-want to know-learned (KWL) techniques could be used depending on the nature of lessons and the level of students.

Reflective practice is one of the best strategies for improving metacognition and it is an authentic way to assess the performance of students. After solving the problems, teachers should instruct students to do reflective writing based on what went well, what did not go well, and what will do differently next time. If possible, teachers should give constructive feedback on individual students' reflective writing and that could be used as a formative assessment. Engaging students to assess reflective writings through self-assessment and peer assessment could improve metacognition and could be enjoyable for mathematical problem solving.

Research points out that stress, frustrations, embarrassment, and boredom lead to aversive attitudes towards learning mathematics (Mager, 1984). Thus, teachers should create a positive learning environment in which the development of students' thinking is encouraged, planning is shared between each other, monitoring, and evaluation is ongoing. Moreover, such an environment should involve teacher modeling and explanation, communicating students to students and students to teacher, students' discussions, teacher's encouragement and support, constructive feedback, and emotional support.

Additionally, knowing students' beliefs, emotions, and behaviors towards mathematics could help teachers to design the lessons more effectively. Possibly, mathematics teachers should explore the attitude of students towards mathematics through attitude inventory questionnaires at the beginning of the school year.

In the present study, the single-group pretest/posttest design, one of the designs of quasi-experimental research was adopted. Thus, in further study, a true experimental study should be conducted to get more valid findings. In addition, the development of metacognitive skills across different age groups and/or the correlation between metacognitive skills, students' attitudes, and achievement in problem solving should be studied.

Conclusion

According to the findings, the proposed metacognitive teaching model supported the positive effects on problem solving behaviors of students, and supported positive effects on their attitude towards problem solving, and twenty-first century skills. Similarly, mathematics teachers advocated that the proposed model had effects on the problem solving behaviors of their students, and positive effects on students' attitude towards mathematics and twenty-first century skill. Additionally, the proposed model had positive effects on their mathematics teaching approaches and it was relevant for teaching problem solving for all Grades at the Middle School level. Therefore, this study pointed out that integrating metacognition in teaching problem solving is one of the reasonable solutions for negative attitudes towards problem solving due to lack of control and monitoring of the solution process.

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